Lesson 20: Four Interesting Transformations of Functions

Classwork

Opening Exercise

Fill in the blanks of the table with the appropriate heading or descriptive information.

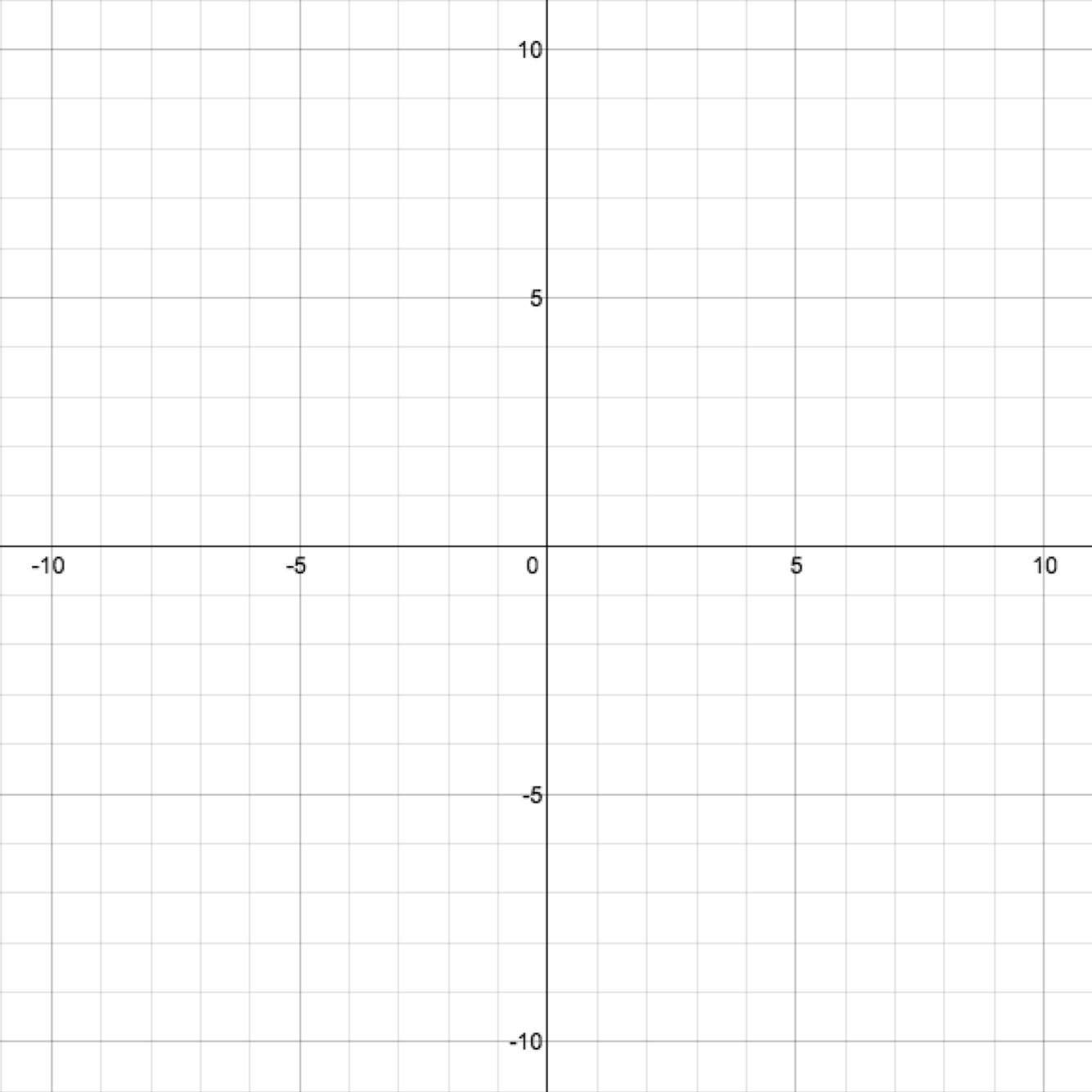
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Graph of | Vertical | | | Horizontal | | |
| Translate |  |  | Translate up by units |  |  | Translate right by units |
|  | Translate down by units |  |  |
| Scale by scale factor |  |  |  |  |  | Horizontal stretch by a factor of |
|  | Vertical shrink by a factor of |  |  |
|  | Vertical shrink by a factor of and reflection over -axis |  | Horizontal shrink by a factor of and reflection across -axis |
|  |  |  | Horizontal stretch by a factor of and reflection over -axis |

Example 1

A transformation of the absolute value function, is rewritten here as a piecewise function. Describe in words how to graph this piecewise function.

Exercises 1–2

1. Describe how to graph the following piecewise function. Then graph below.



1. Using the graph of below, write a formula for as a piecewise function.



Example 2

The graph of a piecewise function is shown. The domain of is , and the range is .

* 1. Mark and identify four strategic points helpful in sketching the graph of .

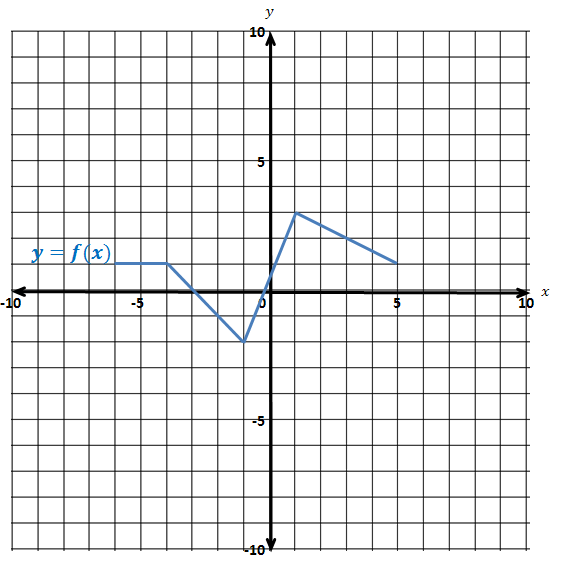


* 1. Sketch the graph of and state the domain and range of the transformed function. How can you use part (a) to help sketch the graph of ?
  2. A horizontal scaling with scale factor of the graph of is the graph of . Sketch the graph of and state the domain and range. How can you use the points identified in part (a) to help sketch ?

Exercises 3–4

1. How does the range of in Example 2 compare to the range of a transformed function , where , when ?
2. How does the domain of in Example 2 compare to the domain of a transformed function , where   
   , when ? (Hint: How does a graph shrink when it is horizontally scaled by a factor ?)

Problem Set

1. Suppose the graph of is given. Write an equation for each of the following graphs after the graph of has been transformed as described.
   1. Translate 5 units upward.
   2. Translate 3 units downward.
   3. Translate 2 units right.
   4. Translate 4 units left.
   5. Reflect about the -axis.
   6. Reflect about the -axis.
   7. Stretch vertically by a factor of 2.
   8. Shrink vertically by a factor of .
   9. Shrink horizontally by a factor of .
   10. Stretch horizontally by a factor of .
2. Explain how the graphs of the equations below are related to the graph of .
   1. ****
3. The graph of the equation is provided below. For each of the following transformations of the graph, write a formula (in terms of for the function that is represented by the transformation of the graph of . Then draw the transformed graph of the function on the same set of axes as the graph of .
   1. A translation units left and units up.
   2. A vertical stretch by a scale factor of .
   3. A horizontal shrink by a scale factor of .
4. Reexamine your work on Example 2 and Exercises 3 and 4 from this lesson. The questions in (b) and (c) of Example 2 asked how the equations and could be graphed with the help of the strategic points found in (a). In this problem, we investigate whether it is possible to determine the graphs of and by working with the piecewise-linear function directly.
   1. Write the function in Example 2 as a piecewise-linear function.
   2. Let . Use the graph you sketched in Example 2b of to write the formula for the function as a piecewise-linear function.
   3. Let . Use the graph you sketched in Example 2c of to write the formula for the function as a piecewise-linear function.
   4. Compare the piecewise linear functions and to the piecewise linear function . Did the expressions defining each piece change? If so, how? Did the domains of each piece change? If so how?